International Journal of Scientific & Engineering Research, Volume 4, Issue 4, April-2013
ISSN 2229-5518

Review on Fatigue-Crack Growth and Finite Element Method

Nirpesh Vikram, Raghuvir Kumar

Abstract— This article reviews the literature on Crack Propagation in fatigue and its numerical analysis using FEM published since 19th century and identifies new research lines. Review shows that fatigue life has been considered to be composed of three phases: **(1)** Crack initiation **(2)** Crack Propagation **(3)** Final Failure. Mechanism of Crack initiation, especially quantitative models are still not known. Initially crack length is subcritical and the crack is not dangerous. In some next cycles, the crack propagates to acquire critical length and then conventional fracture mechanics phenomenon like G_{IC}, K_{IC}, J_{IC}, **CTOD** etc come in to picture. A large number of empirical and semi empirical fatigue crack growth laws have been proposed for many materials through experiments, numerical and analytical methods like FEM (Finite Element Methods). Summarization on crack propagation that the crack closure is affected by the material properties like yield stress "σ_y", fracture stress "σ_f", maximum intensity factor "k_m", stress range "$\triangle\sigma$", crack length "**a**", strain hardening component "**n**", working environment and geometry is presented in this paper.

Index Terms—Fatigue-Crack, Crack Propagation, Crack initiation, Fatigue loading, Crack Growth, Stress intensity Factor, Effective Stress

NOMENCLATURE

Greek Symbols	Description
α	A Variable factor
σ	Normal Stress
σ_{avg}	Average (mean) stress in a Cycle
σ_a	Threshold Stress
σ_m	Maximum Stress in a Cycle
σ_n	Minimum Stress in a Cycle
σ_o	Optimum Stress
σ_p	Stress amplitude in a Cycle
σ_u	Ultimate Stress
σ_f	Fracture Stress
σ_y	Yield Stress
σ_o	Crack Opening Stress
σ_{cl}	Crack Closing Stress
$\triangle\sigma$	Stress Range
$\triangle\sigma_{eff}$	Effective Stress Range
U	Stress Intensity Ratio

English Symbols	Description
a	Crack length
A	A constant
B	Specimen thickness
C	Constant of crack growth equation
da/dN	Crack growth rate
D	A constant
E	Young's modulus of elasticity
K	Stress intensity factor
K_c	Fracture toughness of the material
K_m	Maximum stress intensity factor of a cycle
K_n	Minimum stress intensity factor of a cycle
K_o	Optimum stress intensity factor of a cycle
K_t	Threshold stress intensity factor
$\triangle K$	Stress intensity range
$\triangle K_e$	Effective stress intensity range
m	Exponent of crack growth rate equation
n	Exponent of crack growth rate equation
N	Number of cycles
N_f	Number of cycles to failure
N_P	Number of readings in a set of readings

P	A ratio $\triangle\sigma/\sigma_y$
P	Simple load
P_a	Average load in a cycle
P_m	Maximum load in a cycle
P_n	Minimum load in a cycle
$\triangle P$	Load range in CAL cycle
R	Stress ratio in CAL cycle (P_m/P_m)
W	Width of the specimen

1 INTRODUCTION

Failures of components and structures over years have encouraged the researchers to perform the various failure studies. In general failure of the components is results of two most common reasons one is fatigue loading and other one is effect of working environment in which the component is working like temperature the most common factor for environment affected failure [19]. In real life there are mostly complex loading conditions in which the components work but at the time of analysis whether it can be experimental, analytical or numerical we consider the ideal loading condition to get the solutions easily or to form some empirical formulas. Fatigue is the most common cause of crack initiation and crack growth to critical size [16], at which sudden fracture takes place.

It was realized that crack extension takes place due to stress concentration at the crack tip and due to failure of material during cyclic loading; an effort has been made to relate the crack growth with stress intensity factor "K" at the crack tip. A well established relationship was given by Paris and Erdogan [11] and takes the following form:

$$\frac{da}{dN} = C(\Delta K)^m \qquad (1)$$

Where C, and m depend on material, specimen geometry and loading. It is found that for different values of stress ratios, R, for the same material a large deviation in data was obtained from the curve fitted by eq.(1). The use of the range of cyclic stress intensity factors to describe fatigue crack growth rate is based on the assumption that the crack tip starts to open as soon as load is completely relaxed. In 1968 on the basis of results of experiments, Elber [14] predicted that cyclic plasticity gives rise to the development of residual plastic deformation in the vicinity of the crack tip causing the fatigue crack to close under a positive load. He described this as crack closure phenomenon and suggested that the fatigue crack growth can occur only during the portion of the loading cycle in which the crack is fully open.

• ***Nirpesh Vikram*** *is currently pursuing Ph.D. program in Department of Mechanical Engineering,* ***BBD University****, Lucknow,* ***India****, Mob: 09559996669. E-mail: nirpesh.vikram@gmail.com*
• ***Prof (Dr.) Raghuvir Kumar*** *is currently working as Professor in Department of Mechanical Engineering in* ***BBD University****, Lucknow,* ***India****, Mob: 09044922718. E-mail: bbd_rkg@yahoo.com*

Based on this suggestion, an effective stress range is defined:

$$\triangle\sigma_{eff} = \sigma_m - \sigma_o \ (\text{or } \sigma_{cl}) \qquad (2)$$

The ratio of $\triangle\sigma_{eff}$ to the total stress range ($\triangle\sigma$) is defined as the stress intensity range ratio, U, and is given by

$$U = \frac{\triangle\sigma_{eff}}{\triangle\sigma} = \frac{\sigma_m - \sigma_o \ (\text{or } \sigma_{cl})}{\sigma_m - \sigma_n} \qquad (3)$$

Elber [15] further suggested that the crack growth relationship be written in the following form:

$$\frac{da}{dN} = C(\Delta K_{eff})m = C(U\Delta K)^m \qquad (4)$$

The crack propagation equation is written in terms of "ΔK_{eff} , instead of "ΔK". the factors which have been reported to influence U are stress intensity range ($\triangle\sigma$), material properties (σ_y , σ_f) , crack length (a) and stress ratio R. In the work of Elber [15], however, U is shown to depend only on stress ratio R. Many laws are available which give crack growth rate as a function of $\triangle$K and material properties. In this regards many other researchers [1, 2, 7, 16, 17, 18, 19, 21, 22, 25, 32, 37, 44] had given their contribution to formulate the crack growth but till today there is no generalized theoretical formulations is there for crack growth that is applicable for all type of materials. Variations in types of cracks and loading conditions are other reasons so that there is none of the generalized formulations for crack growth. All the formulations are empirical or semi empirical. In all kind of numerical approaches the best known and mostly accepted approach is Finite Element Method (FEM) [23, 24, 26]. This approach is widely accepted approach by the research community due to its approximate result giving ability. Due to the variations in loading conditions and micro structure of materials it is quite difficult to form a generalized formulation. Generalized formulation needs some more and more experimental and analytical analysis on different materials and different loading conditions. This literature survey is also an initiative in this regard.

2 TERMINOLOGIES

2.1 TYPES OF FATIGUE:

There are three commonly recognized forms of fatigue:

- High cycle fatigue (HCF)
- Low cycle fatigue (LCF)
- Thermal mechanical fatigue (TMF)

2.1.1 HIGH CYCLE FATIGUE (HCF)

The principal distinction between HCF and LCF is the region of the stress strain curve where the repetitive application of load (and resultant deformation or strain) is taking place.

HCF is characterized by low amplitude high frequency elastic strains. An example would be an airfoil subjected to repeated bending. One source of this bending occurs as a compressor or turbine blade passes behind a stator vane. When the blade emerges into the gas path it is bent by high velocity gas pressure. Changes in rotor speed change the frequency of blade loading. The excitation will at some point match the blade's resonant frequency causing the amplitude of vibration to increase significantly. To clarify this concept we need to return to the stress strain curve. When a tuning fork is struck it vibrates at its resonant frequency. As the beams of the fork bend back and forth at hundreds of cycles per second the amplitude of the bending results in strains that are confined to the elastic portion of the stress strain curve. As the vibrations die down and stop the fork returns to its original shape. Only elastic strains have occurred so no permanent deformation has taken place. The tuning fork can endure tens of millions of cycles under these conditions but eventually it will fail due to HCF. Empirical parameters for a lot of materials has been determined like Marin factors and Fatigue Strengths in HCF Both infinite or finite fatigue life is possible and can be analyzed so that it is easy to use for design applications. If loads are fluctuating in a pseudo-random way, HCF methods can yield non-conservative results.

2.1.2 LOW CYCLE FATIGUE (LCF)

LCF is the mode of material degradation when plastic strains are induced in an engine component due to the service environment. LCF is characterized by high amplitude low frequency plastic strains. If we pull the beams of the tuning fork apart until they are permanently bent we have imparted one half of an LCF cycle. The act of permanently bending means that we have exceeded the elastic limit point on the stress strain curve and have crossed over into the plastic region. Forcing the beams back into the original position will require them to bent or "yielded" thereby completing one LCF cycle. The tuning fork can endure only a very few of these cycles before it will fail due to LCF. In a turbine blade these large strains occur in areas of stress concentration. Most turbine blades have a variety of features like holes, interior passages, curves and notches. These features raise the local stress level to the point where plastic strains occur. Turbine blades and vanes usually have a configuration at the base referred to as a dove tail or fir tree. This feature is used to attach the blade to the turbine disk. As engine rotational speed increases centrifugal forces result in local plastic strains at the attachment surfaces resulting in LCF damage.

2.1.3 THERMAL MECHANICAL FATIGUE (TMF)

In the case of TMF (present in turbine blades, vanes and other hot section components) large temperature changes result in significant thermal expansion and contraction and therefore significant strain excursions. These strains are reinforced or countered by mechanical strains associated with centrifugal loads as engine speed changes. The combination of these events causes material degradation due to TMF.

2.2 FATIGUE LOAD

There are two types of fatigue load on which all the research have been done:

2.2.1 CONSTANT AMPLITUDE LOAD

Mathematical interpretation of constant amplitude loading with max stress σ_{max} and minimum stress σ_{min} with the stress range $\triangle\sigma$ given by:

$$\triangle\sigma = \sigma_{max} - \sigma_{min} \tag{5}$$

$$K_{max} = f(a/w)\,\sigma_{max}\sqrt{\pi a} \tag{6}$$

$$K_{min} = f(a/w)\,\sigma_{min}\sqrt{\pi a} \tag{7}$$

Where f(a/w) is the geometric factor for crack length "a" and component width "W".

2.2.2 VARIABLE AMPLITUDE LOAD

Almost all structures and machines are subjected to variable amplitude loading in real life. Due to variation in nature of loads from one kind of application to another they do not follow the Gaussian distribution. Dependent of application statistical method is opted to determine the root mean square value of "ΔK". So if Paris Law [11] is chosen then it becomes:

$$\frac{da}{dN} = C(\Delta K_{rms})^m \tag{8}$$

2.3 S-N CURVE

An empirical relation is determined between applied stress (peak value of the fluctuating load) and number of cycles N required to cause the failure. The relation is known as S-N curve also called the Wohler curve. This is a sigmoidal curve by shape and have been in use for more than a century are still being used by conventional designers and researcher. S-N curve have certain limitations it adopts black box approach and it does not explore the mechanisms of failure. It does not show difference between initiation life and propagation life; only over all fatigue life is taken into account. After testing specimens at different amplitudes of loading, the S-N curve. It represents the number of cycles or life to failure against the stress amplitude s_a. Failure can be defined as fracture or crack initiation. Different stress ratios lead to different S-N curves. We often define the S-N curve for a loading at R = -1. In the following Fig., a typical S-N curve is shown. We observe three different zones on the S-N curve.

The low cycle fatigue is related to the number of cycles from 10^2 to approximately 10^3 or 10^4. Stresses are close to the ultimate tensile Stress σ_u. Some macroscopic plastic deformation appears. In the high cycle fatigue for finite life, the number of cycles goes from 10^4 to approximately 0^5 cycles. The last part is related to weak stresses and infinite fatigue life. The fatigue phenomenon can appear after a long run of loading with an amplitude close to the threshold σ_a, or may even never happen. The scatter is very high in this region. The threshold σ_a represented in Fig 1 is called the fatigue limit. It is usually defined at around 10^6, 10^7 cycles.

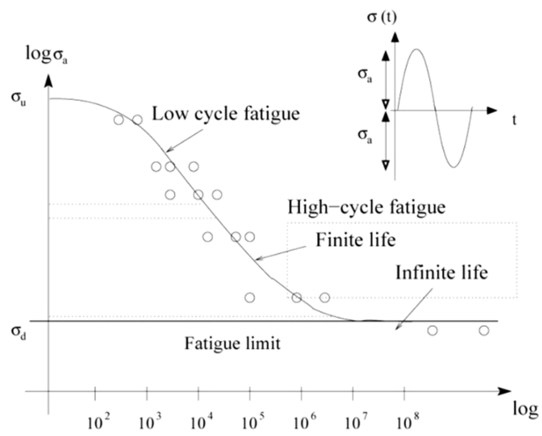

Fig 1: S-N Curve [79]

2.4 CRACK INITIATION [17, 21]

Crack initiation can be seen to occur at the tip of an existing crack or at some point of a free surface. Crack grows in all applied load cycle so that da/dN becomes important parameter. Initially da/dN is extremely small. ΔK increases as the crack grows and da/dN becomes very large. Crack growth divided in to three regions on growth curve plotted of fatigue crack growth on log-log scale (da/dN Vs ΔK) as shown the below figure. Where we can observe there is no crack initiation if ΔK is smaller than the ΔK_{th} .where ΔK_{th} depends on the material properties and stress ratio R.

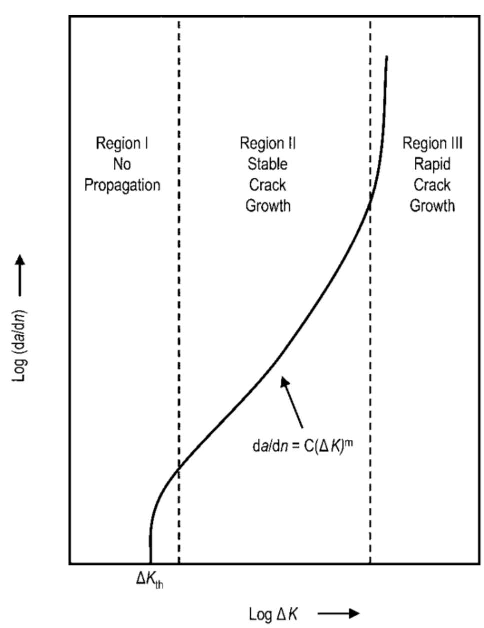

Fig 3: Crack Growth Curve [78]

2.5 CRACK PROPAGATION

There are three modes of crack propagation in which crack propagates and lead to the fracture

- **Mode I** or Opening Mode: displacement is normal to crack surface
- **Mode II** or Shearing Mode or Sliding Mode: displacement is normal to the crack front
- **Mode III** or Tearing Mode: displacement is parallel to crack front

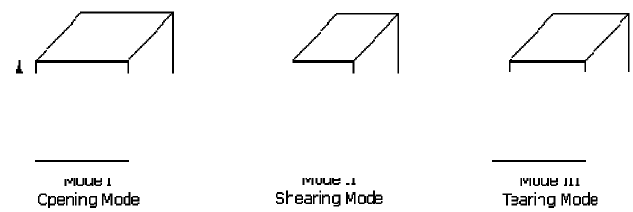

Fig 3: Fracture Modes

2.6 CRACK GROWTH PARAMETRES

Since so many years a considerable amount of work has been done to perform the studies on crack propagation rate under constant amplitude loading and variable amplitude loading. Most of these studies are conducted in terms of development of crack growth rate model. Some important models are presented as follows:

- Energy Release Rate
- Stress Intensity Factor (SIF)
- J-Integral
- Crack Tip Opening Displacement (CTOD)

2.6.1 ENERGY RELEASE RATE

This model is given by Griffith [5] in early 1920, he worked on (Glass) brittle material and gives a phenomenon of Energy Release rate he shows that when a crack initiates it releases energy and if this energy value is equal to or greater than strain energy crack propagates. But later this was seen that this phenomenon is good for brittle but not for ductile materials. Mathematical interpretation of the Griffith ′ s approach says that in any component ′ s crack to become a critical crack that will be responsible to failure the following relationship should be true where $E_R$is representing Energy release Rate which is known as “G” after the name of Griffith and Es represents Energy required to advance per unit length by the crack and that is known as crack Resistance “R”.

$$\frac{dE_R}{da} \geq \frac{dE_S}{da} \quad (9)$$

$$G \geq R \quad (10)$$

2.6.2 STRESS INTENSITY FACTOR (SIF)

Irwin [12] introduced this variable and used the symbol “K” after the name of his research team mate Kies. For Mode I crack propagation

$$K_I = \sigma \sqrt{\pi a} \quad (11)$$

It is well established that the large stresses produced due to stress concentration at the crack tip are responsible for crack growth.

Stress intensity range ΔK at the crack tip is a dominant parameter. Though, ΔK is a well established parameter and its contribution is well known in crack growth curves. Its use has introduced certain amount of discrepancy between actual and predicted results. The reason behind it is that derivation of ΔK depends on equilibrium equations considering only material properties based on linearly related equations. In short ΔK does not consider the effect of non-linear behavior of the material. In addition to nominal stress, crack length and specimen width. The material properties like yield stress, fracture stress, and strain also affect the crack growth rate in any cycle.

2.6.3 J-INTEGRAL (J-INTEGRAL)

J- integral was first applied by Rice [14] for plane problems. Like other parameters (G and K), the J- integral is also a parameter to characterize a crack. In fact G is a special case of J integral. G is usually applied only to linear elastic materials, whereas the J-integral is not only applicable to linear and non-linear elastic materials but is very useful to characterize materials, exhibiting elastic-plastic behavior near crack tip. Mathematical interpretation of J is:

$$J=\int \left(W dx_2 - T_i \frac{\partial u_i}{\partial x_i}\right) \quad (12)$$

(on any path chosen within the body of the specimen)

Where, $$W=\int \sigma_{ij}\, d\varepsilon_{ij} \quad (13)$$

This is path independent, for linear elastic bodies, the J-integral represents energy release rate and is same as G.

2.6.4 CRACK TIP OPENING DISPLACEMENT (CTOD)

This is another type of parameter to characterize a crack. This can be used for both type of linear fracture mechanics (LEFM) and elastic-plastic fracture mechanics (EPFM). Wells [16] was given its mathematical formulation. It was realized that j-integral can also be used for EPFM after a decade of CTOD formulation. For Opening Mode of Crack following relationship exists:

$$CTOD = \frac{K_I^2}{E\sigma_{ys}} = \frac{G_I}{\sigma_{ys}} \quad (14)$$

2.6.5 FATIGUE CRACK PROPAGATION

Ever since it was realized that the crack extension takes place due to stress concentration at the crack tip and the failure of the material during cyclic loading is due to accumulated crack growth in several thousands of cycles during the life span of the specimen, an effort was made to relate crack growth rate with the stress intensity factor at the crack tip. Though the above physical basis has limitations for elasto-plastic material due to presence of large plastic deformation at the crack tip. Paris and Erdogan [11] established a relationship which is expressed as equation (15):

$$\frac{da}{dN} = C(\Delta K)^m \quad (15)$$

After Paris and Erdogan [11] relationship, a sudden surge in the activity occurred for finding out this form of relationship by evaluating the constants C and m for different materials. A large number of data show a large variation in the values of C and m for different materials. These values are also found to change with different loading conditions. Table (2.1) shows some typical values of C and m for different materials [21].

Table 1: Values of constants in crack growth rate equation $\frac{da}{dN} = C(\Delta K)^m$ For ΔK expressed in kg/mm$^{3/2}$

S.No	Material	C	M
1	**Carbon and alloy steels**		
	ASTM A 36 (Plate)	2.14×10^{-11}	3.0
	AISI 4330 (Plate)	1.0×10^{-9}	2.25
2	**Stainless Steel**		
	304 (Plate)	1.30×10^{-11}	3.25
	304N (Plate)	8.93×10^{-12}	3.05
3	**Alluminium Alloys**		
	2024-T6 (Plate)	1.78×10^{-14}	3.0
	7075-T6 (Plate)	2.97×10^{-14}	3.0
4	**Titanium Alloys**		
	6Al-4V (Plate)	7.60×10^{-11}	3.34

The use of stress intensity range, ΔK was found by Paris and Erdogan [11], it is based on the concept of linear elastic fracture mechanics. It is found that for the different values of R for the same material, a large deviation in data is obtained from the curve fitted by equation (15).

Elber [15] in the early 1970's gave the concept that the load responsible for the crack extension is only a part of nominal load range. This is defined by U effective stress intensity range ratio, which is given by the following equation:

$$U = \left(\frac{\Delta K_\theta}{\Delta K}\right) = \frac{(K_m - K_o)}{K_m - K_n} = \frac{\sigma_m - \sigma_o}{\sigma_m - \sigma_n} = \frac{\left[1-\left(\frac{\sigma_o}{\sigma_m}\right)\right]}{1-R} \quad (16)$$

He suggested that ΔK used in Paris-Erdogan equation should be replaced by effective stress intensity range, ΔK_Θ, thus fatigue crack growth rate equation can be represented by:

$$\frac{da}{dN} = C(\Delta K_\theta)^m = C(U\Delta K)^m \quad (17)$$

After the introduction of this concept by Elber [14], a large amount of work has been done for finding U as a function of stress ratio R and K_m. attempts have been made to relate U with material properties also. Some expressions on the stress intensity range ratio are given in table (2)

Table 2: Values of U effective stress intensity range ratio in equation $\frac{da}{dN} = C(\Delta K_\theta)^m$ for ΔK_θ expressed in kg/mm$^{3/2}$

S.no	Materials	Researchers	U=f(R, ΔK or K)
1.	RA, Ti-6	Katcher&	U=0.73+0.82R

	Al-4V	Kaplan [62]	
2.	2219-T851	Katcher& Kaplan [62]	U=0.68+0.19R
3.	2024-T3, Al-Alloy	Schijve[63]	$U=0.55+0.35R+0.1R^2$
4.	2024-T3, Al-Alloy	Elber [64]	U=0.5+0.4R- 0.1<R<0.7
5.	Steel A & C	Maddox [65]	U=0.75+0.25R

As we know, the fatigue crack propagation in a specimen is divided in three regions fig 3. Forman, Karney and Engle [68] paid attention to the third stage, when the fatigue crack propagation rate becomes high and the specimen is in final stage of breaking. They considered that a sharp increase in propagation rate is caused when maximum stress intensity factor becomes close to its critical value, which is related to the fracture toughness of the material.

$$K_m \rightarrow K_c \frac{da}{dN} = infinite \tag{18}$$

Using stress ratio K_m is given by

$$K_m = \frac{\Delta K}{1-R} \tag{19}$$

Considering equation (18) and (19) they [66] modified equation (15) in the form

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{[(1-R)K_c - \Delta K]} \tag{20}$$

The equation (20) fits well on the crack growth rate data obtained by authors on Al -alloys. It however generality as pointed out by Pearson [17].

Pearson proposed that it is not absolutely necessary for the fatigue crack propagation rate to have a power of 1. He showed some examples, where neither equation (15) nor equation (20) were able to fit the experimental data. He proposed a equation which was able to co-relate the data well.

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{[(1-R)K_c - \Delta K]^{1/2}} \tag{21}$$

Here C and m are material constants.

Walker [17] proposed that for small scale yielding the crack propagation rates should be a function of both stress intensity factor range and maximum stress intensity factor.

$$\frac{da}{dN} = C(\Delta K)^m (K_m)^n \tag{22}$$

C, m and n are material constants.

Various crack growth laws can be divided in different categories depending upon the principles on which they are derived. These laws [14] are given in table (3)

Table 3: Various crack growth laws as given by different researchers.

Nature	Laws	References
Emperical laws	$\frac{da}{dN} = C(\Delta K)^m$	Paris [11]
	$\frac{da}{dN} = C(\Delta K)^m (K)^n$	Walker [17]
	$\frac{da}{dN} = \frac{C(\Delta K)^m}{[(1-R)K_c - \Delta K}$	Foreman[68]
	$\frac{da}{dN} = \frac{C(\Delta K)^m}{[(1-R)K_c - \Delta K]}$	Foreman[68]
Based on deformation ahead of crack tip	$\frac{da}{dN} = C\omega^m \Delta\omega^p$	Erdogan[78]
	$\frac{da}{dN} = C\omega$	Liu[78]
	$\frac{da}{dN} = \Delta\varepsilon_p \Delta\omega$	Tomkins[78]
Based on crack tip geometry	$\frac{da}{dN} = \frac{8}{\sigma\left(\frac{\Delta K}{E}\right)^2}$	Frost & Dixon[78]
	$\frac{da}{dN} = \frac{8}{\pi\left(\frac{\Delta K}{E}\right)^2}$	Pook & Frost[78]
Crack closure concept	$\frac{da}{dN} = C(U\Delta K)^m$	Elber [64]
	$\frac{da}{dN} = \frac{(0.886U)^{1+n}p^m}{500(1-R)^{1+n}}a$ Where $p = \sigma/\sigma_y$	Lal [70]

3 CRACK GROWTH RATE CONSTANTS

As we have seen earlier constants of the models in table (3) are called crack growth rate constants. These crack growth rate constants are found by drawing $log(\Delta K) - log(\frac{da}{dN})$ curves for each set of loadings. Slope of the $log(\Delta K) - log(\frac{da}{dN})$ curves gives us the value of constant m, while the intercept of the curve at Y-axis will give us the value of the constant C. Generally the values of these constants vary material to material. Variation has also been found among different set of loadings i.e. the values of constants C and m varies as the parameters, maximum load, minimum load and load range varies. Researchers tried to co-relate the constants C and m with loading parameters.

Niccollos [66] proposed the relationship between m and C which is given follows.

$$m = A + D\ (log\ C) \tag{23}$$

Here A and D are negative parameters that remain constant for a given class of materials.

A comprehensive study of the equation (23) was carried out by Tanaka et al [67]. These authors resolved the presence of a pivot point (PP), with co-ordinates $(\frac{da}{dN})_p$ and ΔK_p , where all the $log(\frac{da}{dN})$ versus $log(\Delta K)$ straight lines intersected. It is shown in figure (3). In relation to pivot point equation (15) becomes:

$$\frac{da}{dN} = \left(\frac{da}{dN}\right)_p \left(\frac{\Delta K}{\Delta K_p}\right)^m \quad (24)$$

With $\left(\frac{da}{dN}\right) = exp\left(-\frac{A}{B}\right)$ and $\Delta K_p = exp(-\frac{1}{B})$

Bailon et al [68] first suggested a relation between the load ratio and the m-logC relationship. They carried out investigations on aluminium alloys 2024-T3, 2618-AT651 and 7175-T7351, with load ratios from -0.3 to 0.75, and found pronounced effect of R, and co-related the coefficients of equation (23).

With modulus of R

$$A = -1.98 - 6.85\,R \quad (25)$$

$$D = -0.29 - 0.47\,R \quad (26)$$

4 METHODS OF ANALYSIS AND SOME IMPORTANT RESULTS:

4.1 EXPERIMENTAL

In the past various methods have been used for establishing the crack opening and crack closing points. A displacement gauge is usually mounted either across the crack or at the mouth of the notch, and the load displacement curve is taken. The change in slope of the load displacement curve gives an indication of crack opening and closing. Some researchers have also tried ultrasonic and electric potential methods. However, because of difficulties in interpreting the results, the COD method is still considered to be superior to other methods. A review of work on crack closure at CAL reported in the literature using the above method.

Constant amplitude loading (CAL) work is divided in to the following categories:

(i) Dependence of crack closure on stress ratio R;
(ii) Dependence of crack closure on stress ratio R, K_{max} and ΔK;
(iii) Dependence of crack closure on material properties σ_y, n, σ_f,
(iv) Dependence of crack closure on environment and instantaneous crack length;
Classification on the basis of crack closure measurement techniques

4.1.1 EFFECT OF MEAN STRESS ON FATIGUE CRACK GROWTH

$$\mathbf{R} = \frac{\sigma_{min}}{\sigma_{max}} = \frac{K_{min}}{K_{max}} = \text{Stress Ratio} \quad (27)$$

$$R=0,\ R>0$$

4.1.2 ENVIRONMENTAL EFFECT ON FATIGUE GROWTH RATE

There is an overall enhancement of crack growth rate except near the threshold ΔK_{th} can be higher in corrosive environment. The corrosion Products increase the volume of material contributing to the crack closure thus pushing up ΔK_{th}

4.1.3 DEPENDENCE OF CRACK CLOSURE ON STRESS RATIO R:

A large number of research workers have found that for a given material, U is a function of R only and is independent of other parameters. The Elber [64] model is valid for -0.1<R<0.7. Katcher and Kaplan [62] observed no crack closure after a stress raotio of 0.3. Schijve [63] found U as a function of second order polynomial in R. The model is valid for both positive and negative values of stress ratio. Buck found that U and R for various materials tested by many authors.

4.1.4 DEPENDENCE OF CRACK CLOSURE ON STRESS RATIO R, K_{MAX} AND ΔK;

According to some other researcher [69, 70] U depends on K_{max} , R and ΔK. Chand & Garg [71] developed models for U as function of K_{max} , and R.Srivastav and Garg [71]developed a model of for U as a function of ΔK and R. Srivastava and Garg [71]showed that U is a function of R and ΔK and that U tends to increase with increasingΔK. Clark and Cassat [72] found that for specimen thickness of 6.35 and 25.40 mm, U increases with increasing K_{max}, but for a thickness of 12.70 mm it decreases.

4.1.5 DEPENDENCE OF CRACK CLOSURE ON MATERIAL PROPERTIES σ_Y, n, σ_f,

Some researchers found that U is a function of stress ratio and material properties. Newman, Bell and Creager, Kumar and Garg developed models for U as a function of material properties like stress σ_y , cyclic hardening exponent (n) etc. Elber [64], Schijve and Kumar and Garg found that crack closure load is less in comparison to as received material due to high yield strength.

4.1.6 DEPENDENCE OF CRACK CLOSURE ON ENVIRONMENT AND INSTANTANEOUS CRACK LENGTH;

Bachmann and Munz, Irving, Homma and Nakazwa , Morris and James and Ho found changes in U with gauge location along the crack line. Schijve [63], Srivastava, Lal, Kumar and Garg found that crack closure load is independent of instantaneous crack length. Schijve and Arkema also showed that crack closure load is the same in all three environment (Vacuume, air, salt water). Buck showed that lower crack closure loads in moist air in comparison to a dry atmosphere. Schijve [63] found lesser crack closure load in thick material. Kumar and Garg showed that the crack closure equation is valid for both SEN and centrally notched specimen.

4.2 FINITE ELEMENT METHOD

Finite element Method is one of the numerical methods to obtain an approximate solution to many of the fracture mechanics problems. Today this method has become so powerful due to high end computers are available. The basic approach to solve the problem by this method is the domain of the problem is discretized in to number of sub domains called elements which are connected with other at points called nodes. All the variables are approximated piecewise, so that they are represented in each element by simple polynomials. The coefficient of the polynomial equivalently expressed as nodal values of the variables are determined such that governing equations and boundary conditions are satisfied in the best possible manner. This approximation method may be variational method or a weighted residual approach.

There two methods to determined fracture parameters:

- Direct Methods to determine fracture parameter
- Indirect Method to determine fracture parameter

4.2.1 DIRECT METHODS TO DETERMINE FRACTURE PARAMETER

In this kind of method we assume stress, strain field in a 2D crack problem by using 3 or 6 noded triangular elements, 4 noded iso parametric elements. Now for determination of fracture parameter K_I, K_{II}, near the crack tip we can use the expression of displacement and stress near the crack tip. Watwood [75] gave some studies results with this method on central cracked specimen.Chan et al. [77] determined K_I value with coarse mesh. Very near crack tip the solution of FEM become in accurate due to its inability to model singular nature of stresses accurately.

4.2.2 INDIRECT METHOD TO DETERMINE FRACTURE PARAMETER

There are some methods in which no need to use direct formula of crack tip stress. This gives improved results than direct methods. Some important methods also reviewed as follows:

4.2.2.1 J-INTEGRAL METHOD

In this method the path of integration for j-integral is taken along the nodes on the element ' s edges. And the strain energy density values at nodes are obtained an extrapolation of the values at Gauss points within the elements. Chan et al. [77] applied this to analyze a compact test specimen.

4.2.2.2 ENERGY RELEASE RATE METHOD

Watwood [75] applied this method to analyze a center cracked panel of finite size. And the obtained results further compared with the results obtained by Isida [76] and error was just 2%. This method can also be applied for 3D case.

4.2.2.3 STIFFNESS DERIVATIVE METHOD

In this method we calculate the change in potential energy in finite element analysis that uses the change in stiffness of the plate for two configuration of the crack.

4.2.2.4 SINGULAR ELEMENT METHOD

All above were not capable to model the large stress gradient at the crack tip that is theoretically infinite stress at crack tip. Singular elements are special elements which have the interpolation function to model the singularity at the crack tip.

4.2.2.5 BARSOUM ELEMENT METHOD

Barsoum [73] introduced a new method as quarter- Point element (Barsoum Element) technique. In this method 6 noded tringle or 8 noded isoparametric quadrilateral element are used.

4.2.2.6 EXTENDED FINITE ELEMENT METHOD

In this modern and most effected method a new element called enriched element is introduced at the crack tip and outside of the crack tip conventional element is place. Gifford and Hilton [74] was introduced this method and it gives more accurate results than other methods. Now almost all FEM based programs like Abaqus® and ANSYS® etc also accepted this method and worldwide research and industrial filed this method is widely accepted.

5 CONCLUSIONS

The literature review provided us the following important informations:

1. Fatigue crack growth rate equation expressed in terms of the Stress Intensity Factor range ΔK depend on the R-ratio.
2. From literature review we got the following types of crack growth rate equations.

$$\frac{da}{dN} = C\,(\Delta K)^m$$

$$\frac{da}{dN} = C(\Delta K_\theta)^m$$

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{[(1-R)K_c - \Delta K]}$$

$$\frac{da}{dN} = C(\Delta K)^m (K_m)^n$$

   Where $\Delta K_\theta = U\Delta K$, for a particular material U is found to be a function of loading conditions. The constants m, n and C are material constants depending upon loading conditions.
3. Some researchers [64] have found that U is a function of R only, and is independent of other parameters for a material.
4. Some others [70] have found U to increase with R in almost all cases but it also depends upon K_m. In some cases U is found to increase with K_m and in others it is found to decrease with K_m.
5. Results of U are found to be influenced by measurement technique and the material being investigated.
6. Specimen geometry and material properties are found to affect crack growth rate.
7. The power coefficients C and m depends upon loading conditions. Influence of load ratio increases with the value of m, the power coefficient of Paris law [11].
8. Crack closure was still considered to be the leading mechanism to arrive at effective stress range. However, σ_{op} was no longer obtained from an empirical function.
9. This has been seen that Extended Finite Element Method is the best approach [74] to analyze a crack

growth there is a lot of work required to be done on different materials on which experimental work has already done for the comparison with result obtained by XFEM analysis.

10. This has been seen that there is no work found on effect of “n (work hardening exponent)” and “C (material constant)” in fatigue crack growth so there is a relationship required that shows the effect of value of “n” and “C”on the fatigue growth rate.

6 REFERENCES

[1] Braithwaite, F. “On the Fatigue and Consequent Fracture of Metals”, Minutes of Proc. ICE (1854) 463

[2] Ewing, J.A.; Humfrey, J.C.W. “The Fracture of Metals under Repeated Alternations of Stress”, Philosophical Transactions, Royal Society London, CC (1903) 241

[3] Basquin, O.H., “The Exponential Law of Endurance Tests”, Proc. Annual Meeting ASTM, 10 (1919) 625

[4] Inglis, C.E. “Stresses in a Plate due to the Presence of Cracks and Sharp Corner.”, Transactions of the Institute of Naval Architects, 55 (1913) 219

[5] Griffith, A.A. “The Phenomena of Rupture and Flow in Solids.” Philosophical Transactions A, 221 (1920) 163

[6] Neuber, H., “Theory of Stress Concentration for Shear-Strained Prismatical Bodies with Arbitrary Nonlinear Stress Strain Law.” Journal of Applied Mechanics, 28 (1961) 544

[7] Orowan, E. “Theory of the Fatigue of Metals.” Proc. Royal Society A, 171 (1939) 79

[8] Orowan, E. “Fracture and Strength of Solids.” Reports on Progress in Physics, 12 (1948) 185

[9] Peterson, R. E., “Stress Concentration Design Factors”, John Wiley, New York.(1953)

[10] Coffin, Manson, “Introduction to high-temperature low-cycle fatigue.” Experimental Mechanics, v8, n5, May (1955), p218-224.

[11] Paris, P.C., Erdogan, F.: A Critical Analysis of Crack Propagation Laws, Journal of Basic Engineering, 85 (1963) 528

[12] Irwin, G.R.: Fracture, Handbuch der physic, S. Flugge(ed.), Springer-Verlag, Berlin, Vol. VI, pp. 551-590 (1958)

[13] Burdekin, F.M.; Stone, D.E.W.: The Crack Opening Displacement Approach to Fracture Mechanics in Yielding Materials, Journal of Stress Analysis, 1 (1966) 145

[14] Rice, J.R. A path Independent Integral and Approximate Analysis of Strain Concentration by Notches and Cracks, Journal of Applied Mechanics, 35, pp.379-386(1969)

[15] Elber, W.: Fatigue Crack Propagation: Some Effects of Crack Closure on the Mechanism of Fatigue Crack Propagation under Cyclic Tension Loading, PhD Thesis, University of New South Wales (1968)

[16] Wells, A.A. “Application of Fracture Mechanics at and beyond general yielding, “British Welding Journal,10, pp. 563-570(1963)

[17] Walker, K., "The Effect of Stress Ratio During Crack Propagation & Fatigue for 2024-T3 and 7075-T6 aluminum", ASTM STP 462, pp 1-14, (1970)

[18] Barsom, J.M. and McNicol, R.C. “Effect of stress concentration on fatigue crack initiation in HY-130 Steel, ASTM STP 559, American Society for Testing and Materials, Philadelphia, pp. 183-204, (1974)

[19] Pearson, S.: “Initiation of Fatigue Cracks in Commercial Aluminum Alloys and the Subsequent Propagation of Very Short Cracks”, Engineering Fracture Mechanics, 7 (1975) 235

[20] Niccolls, E.H., "A Co-relation for Fatigue Crack Growth Rate", ScriptaMetall 10, pp 295-298, (1976).

[21] Osgood, C.C., “Fatigue Design”, Cranburry, New Jersey, USA, Peragamon Press, (1982)

[22] Chand, S. &Garg. S.B.L. , "Crack Propagation Under constant Amplitude Loading", Engineering Fracture Mechanics, Vol. 21, No. 1, pp 1-30, (1985).

[23] Hertzberg, Richard W., “Deformation and Fracture Mechanics of Engineering Materials, John Wiley & Sons, New York.(1989)

[24] Shiozawa, K.; Matsushita, H. “Crack Initiation and Small Fatigue Crack Growth Behavior of Beta Ti-15V-3Cr- 3Al-3Sn Alloy”, Proc. Fatigue ’96, G. Lutjering, H. Nowack (Eds.), Berlin (1996) 301

[25] Couroneau N, Royer J. “Simplified model for the fatigue crack growth analysis of surface cracks in round bars under mode I.” International Journal of Fatigue (1998);20:711–8.

[26] Lin XB, Smith RA. “Finite element modeling of fatigue crack growth of surface cracked plates.” Part I: The numerical technique. Engineering Fracture Mechanics, (1999) 63, p.503-522.

[27] Nicolas Mooesy, John Dolbowz & Ted Belytschko,“A finite element method for crack Growth without remeshing” International Journal For Numerical Methods In Engineering, Int. J. Numer. Meth. Engng. 46, 131{150 (1999)

[28] T. Chang, W. Guo, “Effects of strain hardening and stress state on fatigue crack closure.” International Journal of Fatigue 21 (1999) 881–888

[29] N. Sukumarz, N. Mooesx, B. Moran & T. Belytschko, “Extended finite element method for three-dimensional crack modeling”. International Journal of Fatigue 21 (2000) 801–823

[30] Tada, H.; Paris, P.C.; Irwin, G.R.: The Stress Analysis Handbook, 3rd edition, ASME Press, New York (2000)

[31] Thomas-Peter Fries & Ted Belytschko. “The extended/generalized finite element method: An overview of the method and its applications”, International Journal For Numerical Methods In Engineering, Int. J. Numer. Meth. Engng (2000); 00:1–6 Prepared using nmeauth.cls [Version: 2002/09/18 v2.02]

[32] F. Taheria, D. Traskb, N. Pegg. “Experimental and analytical investigation of fatigue characteristics of 350WT steel under constant and variable amplitude loadings” Marine Structures 16 (2003) 69–91

[33] Chien-Yuan Hou, “Three-dimensional finite element analysis of fatigue crack closure behavior in surface

flaws", International Journal of Fatigue 26 (2004) 1225–1239

[34] J. Zapateroa, B. Morenoa, A. Gonza´lez-Herreraa, J. Domı´nguezb, "Numerical and experimental analysis of fatigue crack growth under random loading" International Journal of Fatigue 27 (2005) 878–890

[35] Stoyan Stoychev, Daniel Kujawski. "Analysis of crack propagation using ΔK and K_{max}" International Journal of Fatigue 27 (2005) 1425–1431

[36] Sarinova Simandjuntak, Hassan Alizadeh, Martyn J. Pavier, David J. Smith, "Fatigue crack closure of a corner crack: A comparison of experimental results with finite element predictions" International Journal of Fatigue 27 (2005) 914–919

[37] M. Sander, H.A. Richard, "Finite element analysis of fatigue crack growth with interspersed mode I and mixed mode overloads" ,International Journal of Fatigue 27 (2005) 905–913

[38] S. Simandjuntak, H. Alizadeh, D.J. Smith, M.J. Pavier, "Three dimensional finite element prediction of crack closure and fatigue crack growth rate for a corner crack". International Journal of Fatigue 28 (2006) 335–345

[39] B. Li, L. Reis, M. de Freitas, " Simulation of cyclic stress/strain evolutions for multiaxial fatigue life prediction" International Journal of Fatigue 28 (2006) 451–458

[40] Branco R., "Numerical study of fatigue crack growth in MT specimens". MSc Thesis, Department of Mechanical Engineering, University of Coimbra, (2006).

[41] G. Jovicic, M. Zivkovic, N. Jovicic, "Extended Finite Element Method for Two-dimensional Crack Modeling" Journal of the Serbian Society for Computational Mechanics / Vol. 1 / No. 1, (2007) / pp. 184-196

[42] J.E. LaRue, S.R. Daniewicz, "Predicting the effect of residual stress on fatigue crack growth." International Journal of Fatigue 29 (2007) 508–515

[43] H. Alizadeh , D.A. Hills , P.F.P. de Matos, D. Nowell , M.J. Pavier ,R.J. Paynter, D.J. Smith, S. Simandjuntak. "A comparison of two and three-dimensional analyses of fatigue crack closure" International Journal of Fatigue 29 (2007) 222–231

[44] Branco R, Antunes FV. "Finite element modeling and analysis of crack shape evolution in mode-I fatigue Middle Cracked Tension specimens", Engineering Fracture Mechanics, 75, p. 3020-303(2008).

[45] Branco R, Antunes FV, Martins RF (2008a). "Modeling fatigue crack propagation in CT specimen", Fatigue and Fracture of Engineering Materials and Structures, 31, p. 452-465.

[46] J. Huynh, L. Molent , S. Barter, "Experimentally derived crack growth models for different stress concentration factors" International Journal of Fatigue 30 (2008) 1766–1786

[47] Tianwen Zhao, Jixi Zhang, Yanyao Jiang, " A study of fatigue crack growth of 7075-T651 aluminum alloy" International Journal of Fatigue 30 (2008) 1169–1180

[48] Y. Lei, "Finite element crack closure analysis of a compact tension specimen" International Journal of Fatigue 30 (2008) 21–31

[49] Belytschko, Gracie, Ventura, "A Review of Extended/Generalized Finite Element Methods for Material Modeling" (2009)

[50] S. Kalnaus, F. Fan, Y. Jiang, A.K. Vasudevan "An experimental investigation of fatigue crack growth of stainless steel 304L." International Journal of Fatigue 31 (2009) 840–849

[51] Jankowiak, H. Jakubczak, G. Glinka, "Fatigue crack growth analysis using 2-D weight function." International Journal of Fatigue 31 (2009) 1921–1927

[52] F. Romeiro, M. de Freitas, M. da Fonte, "Fatigue crack growth with overloads/underloads: Interaction effects and surface roughness" International Journal of Fatigue 31 (2009) 1889–1894

[53] S. Mikheevskiy , G. Glinka, " Elastic–plastic fatigue crack growth analysis under variable amplitude loading spectra." International Journal of Fatigue 31 (2009) 1828–1836

[54] Zengliang Gao, Baoxiang Qiu, Xiaogui Wang, Y. Jiang b "An investigation of fatigue of a notched member" International Journal of Fatigue 32 (2010) 1960–1969

[55] S. Ismonov , S.R. Daniewicz. " Simulation and comparison of several crack closure assessment methodologies using three-dimensional finite element analysis" International Journal of Fatigue 32 (2010) 1322–1329

[56] J.M. Alegre, I.I. Cuesta, " Some aspects about the crack growth FEM simulations under mixed-mode loading" International Journal of Fatigue 32 (2010) 1090–1095

[57] M. El-Zeghayar, T.H. Topper, K.A. Soudki , "A model of crack opening stresses in variable amplitude loading using smooth specimen fatigue test data for three steels." International Journal of Fatigue 33 (2011) 1337–1350

[58] N. Ranganathan, H. Aldroe, F. Lacroix, F. Chalon, R. Leroy, A. Tougui, "Fatigue crack initiation at a notch" International Journal of Fatigue 33 (2011) 492–499

[59] Chien-Yuan Hou, "Simulation of surface crack shape evolution using the finite element technique and considering the crack closure effects" International Journal of Fatigue 33 (2011) 719–726

[60] John A.R. Bomidi, Nick Weinzapfel, Chin-Pei Wang, Farshid Sadeghi, "Experimental and numerical investigation of fatigue of thin tensile specimen." International Journal of Fatigue 44 (2012) 116–130

[61] I.V. Singh, B.K. Mishra, S. Bhattacharya, R.U. Patil. " The numerical simulation of fatigue crack growth using extended finite element method." International Journal of Fatigue 36 (2012) 109–119

[62] Katcher, M., & Kaplan, M., "Effect of R-Factor & Crack Closure on Fatigue Crack Growth for Aluminium & Titanium alloys", ASTM STP 559, pp 264-282, (1974)

[63] Schijve, J. , "Some formulas for the crack opening stress level", Engineering Fracture Mechanics 14, pp 461-465, (1981)

[64] Elber, W., "The significance of fatigue crack closure ASTM 486, pp 230-242, (1971).

[65] Maddox, S.J., et al., "An investigation of the influence of applied stress ratio on fatigue crack propagation in

ISSN 2229-5518

structural steels" welding institute, research report 72/1978/E, (1978)

[66] Niccolls, E.H., "A Co-relation for fatigue crack growth rate", Scripta Metall 10, pp 295-298, (1976).

[67] Tanaka, K. & Matsuoka, S., " A tentative explanation for two parameters C & m in equation of fatigue crack growth rate", International Journal of Fracture 13, 5, pp 563-583, (1977)

[68] Bailon, J.P., Masourave, J. & Bathias, C., "On the relationship between parameters of Paris Law for fatigue crack growth rate in aluminium alloys" Scripta Metall 11, pp 1101-1106, (1977)

[69] Chand, S. & Garg, S.B.L., "Crack propagation under constant amplitude loading", Engineering Fracture Mechanics, Vol.21, No. 1, pp 1-30, (1985)

[70] Chand, S. & Garg, S.B.L., "Crack propagation under constant amplitude loading", Engineering Fracture Mechanics, Vol.18, No. 1, pp 333-347, (1983)

[71] Y.P. Srivastava and S.B.L. Garg, "Influence on R on effective stress range ratio in crack growth. Engineering Fracture Mechanics." 22, 915-926, (1985)

[72] C.K.Clark and G.C. Cassat, "A study of fatigue crack closure using electric potential and compliance techniques." Engineering fracture mechanics. 9, 675-688, (1977)

[73] Barsoum, R.S. "On the use of isoparametric elements in linear fracture mechanics," International journal for numeric methods in engineering, 10, pp. 25-38.(1976)

[74] Gifford, Jr. L.N. and Hilton, P.D. "Stress intensity factors by enriched finite elements "Engineering Fracture Mechanics, 10, pp.507-496,(1978)

[75] Watwood Jr., V.B. "The finite element method for prediction of crack behavior", Nuclear Engineering and design, 11,pp.323-332, (1969)

[76] Isida , M. " On the tension of a strip with a central elliptical hole, " Transactions of Japanese society of mechanical engineering, 21, pp. 507-518, (1955)

[77] Chan, S.K., Tuba, I.S. and Wilson, W.K., "On the finite element method in linear fracture mechanics, 2, pp. 1-17, (1970)

[78] Fatigue Failures, Failure Analysis andPrevention, Vol 11, ASM Handbook, ASM International, 2002

[79] J. SCHIJVE., "Fatigue of Structures and Materials". Kluwer Academic Publisher, (2001).